Covariant canonical quantization of fields and Bohmian mechanics

Hrvoje Nikolić
Theoretical Physics Division, Rudjer Bošković Institute,
P.O.B. 180, HR-10002 Zagreb, Croatia
e-mail: hrvoje@thphys.irb.hr

February 7, 2008

Abstract

We propose a manifestly covariant canonical method of field quantization based on the classical De Donder-Weyl covariant canonical formulation of field theory. Owing to covariance, the space and time arguments of fields are treated on an equal footing. To achieve both covariance and consistency with standard noncovariant canonical quantization of fields in Minkowski spacetime, it is necessary to adopt a covariant Bohmian formulation of quantum field theory. A preferred foliation of spacetime emerges dynamically owing to a purely quantum effect. The application to a simple time-reparametrization invariant system and quantum gravity is discussed and compared with the conventional noncovariant Wheeler-DeWitt approach.

1 Introduction

One of the main open problems of modern theoretical physics is how to combine the principle of quantum mechanics with the principle of general relativity. One aspect of this problem - the existence of nonrenormalizable infinities in perturbative quantum gravity - seems to be solved by two major approaches to quantum gravity, namely, by string theory [1] and by loop quantum gravity [2, 3, 4]. Nevertheless, neither of these two approaches fully incorporates the principle of general relativity. String theory is a perturbative approach depending on the choice of the background metric, while loop quantum gravity is a canonical approach that does not treat time on an equal footing with space. There are attempts to solve these problems by introducing a background independent M-theory for strings [5] or a spacetime spinfoam formalism for loops [4], but these attempts are not yet fully successful.

An alternative, in a sense more conservative approach is to try to modify the usual canonical quantization rules for fields (where time plays a special role) by introducing *covariant* canonical quantization rules. In fact, several classical covariant canonical formalisms are already known, such as the covariant phase space formalism [6, 7] and various versions of the multimomenta formalism (see, e.g., [8, 9, 10] and references therein). Unfortunately, a satisfying method of quantization based on some of these classical formalisms is still not known. Perhaps, the most towards covariant canonical quantization of fields has been done in [11], where a method of quantization of the classical De Donder-Weyl covariant canonical formalism has been proposed. (For the application to quantum gravity, see [12].) However, this proposal suffers from two problems. First, the quantum formalism introduces a new fundamental dimensional constant, the physical meaning of which is not completely clear. Second, the theory is able to describe the usual time-space asymmetric wave-functional states $\Psi([\phi(\mathbf{x})], t)$, but only if $\Psi([\phi(\mathbf{x})], t)$ is a

product state of the form $\prod_{\mathbf{x}} \Psi(\phi(\mathbf{x}), \mathbf{x}, t)$. The aim of this paper is to propose a different method of quantization based on the classical De Donder-Weyl formalism, such that the problems of the approach of [11] are avoided.

Before starting with the De Donder-Weyl formalism, let us give a few additional qualitative remarks on possibilities for covariant quantization. The classical covariant phase space method [6, 7] rests on the existence of space of classical solutions to the equations of motion. However, in the conventional formulation of quantum field theory, it is not clear what might play the role of an analog of classical solutions. On the other hand, in the deterministic Bohmian formulation of quantum field theory [13, 14, 15, 16], such an analog exists. This suggests that the Bohmian formulation might be a basis for a covariant formulation of quantum field theory. In fact, among various equivalent formulations of nonrelativistic quantum mechanics [17], the Bohmian formulation is conceptually the most similar to classical physics, which again suggests that well-understood classical covariant theories might be covariantly quantized most easily by using the Bohmian formulation. Finally, since space and time should play equal roles in a covariant quantum field theory, in general, one might expect wave functionals of the form $\Psi([\phi(x)], x)$ instead of the usual time-space asymmetric wave functionals $\Psi([\phi(\mathbf{x})], t)$. This implies that time-dependent fields $\phi(x) = \phi(\mathbf{x}, t)$ should play a role in a covariant approach, which might be related to the Bohmian formulation that assigns a deterministic time evolution to the field $\phi(\mathbf{x}, t)$.

Of course, the arguments above have only a heuristic value and are not sufficiently convincing by themselves. However, in the present paper, the heuristic arguments above are turned into much stronger and more convincing arguments. In contrast to the usual approaches to Bohmian mechanics, the Bohmian formulation is *not postulated* for interpretational purposes, but *derived* from the purely technical requirements - covariance and consistency with standard quantum field theory. In this way, a covariant version of Bohmian mechanics emerges automatically, as a part of the formalism without which the theory cannot be formulated consistently. This, together with the results of [18, 19] related to relativistic first quantization, suggests that it is Bohmian mechanics that might be the missing bridge between quantum mechanics and relativity. In addition, we also note that the Bohmian interpretation might play an important role for quantum cosmology [20, 21, 22, 23, 24] and noncommutative theories [25].

The paper is organized as follows. Sec. 2 contains two subsections, one presenting a short review of the classical covariant canonical De Donder-Weyl formalism, while the other presenting a short review of the Bohmian formulation of the conventional canonical field quantization. For simplicity, the results are presented for a real scalar field in flat spacetime. The central section, Sec. 3, combines the results presented in Sec. 2 to formulate a covariant canonical quantum formalism consistent with the conventional canonical quantization of fields in flat spacetime. Sec. 4 contains generalizations that include a larger number of fields and curved spacetime. In Sec. 5, the formalism is applied to a simple toy model with time-reparametrization invariance, as well as to quantum gravity. The discussion of our results is presented in the final section, Sec. 6.

In the paper, we use the units such that the velocity of light is $c = 1$, while the signature of the metric is $(+, -, -, -)$.

2 Preliminaries

2.1 Classical De Donder-Weyl formalism

In this subsection we briefly review the classical covariant canonical De Donder-Weyl formalism. (For more details, we refer the reader to [8, 26] and references therein.) For simplicity, we

present the formalism for one real scalar field in Minkowski spacetime, while the generalizations are discussed in Sec. 4.

Let $\phi(x)$ be a real scalar field described by the action

$$\mathcal{A} = \int d^4x \mathcal{L}, \tag{1}$$

where

$$\mathcal{L} = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - V(\phi). \tag{2}$$

The corresponding covariant canonical momentum is given by the 4-vector

$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \partial^\mu \phi. \tag{3}$$

The covariant canonical equations of motion are

$$\partial_\mu \phi = \frac{\partial \mathcal{H}}{\partial \pi^\mu}, \qquad \partial_\mu \pi^\mu = -\frac{\partial \mathcal{H}}{\partial \phi}, \tag{4}$$

where the scalar De Donder-Weyl Hamiltonian (not related to the energy density!) is given by the Legendre transform

$$\begin{aligned} \mathcal{H}(\pi^\alpha, \phi) &= \pi^\mu \partial_\mu \phi - \mathcal{L} \\ &= \frac{1}{2}\pi^\mu \pi_\mu + V. \end{aligned} \tag{5}$$

Eqs. (4) are equivalent to the standard Euler-Lagrange equations of motion. By introducing the local vector $S^\mu(\phi(x), x)$, the dynamics can also be described by the covariant De Donder-Weyl Hamilton-Jacobi equation

$$\mathcal{H}\left(\frac{\partial S^\alpha}{\partial \phi}, \phi\right) + \partial_\mu S^\mu = 0, \tag{6}$$

together with the equation of motion

$$\partial^\mu \phi = \pi^\mu = \frac{\partial S^\mu}{\partial \phi}. \tag{7}$$

Note that in (6), ∂_μ is the *partial* derivative acting only on the second argument of $S^\mu(\phi(x), x)$. The corresponding total derivative is given by

$$d_\mu = \partial_\mu + (\partial_\mu \phi)\frac{\partial}{\partial \phi}. \tag{8}$$

Note also that (6) is a single equation for four quantities S^μ. Consequently, there is a lot of freedom in finding solutions to (6). Nevertheless, the theory is equivalent to other formulations of classical field theory.

Now, following [11], we consider the relation between the covariant Hamilton-Jacobi equation (6) and the conventional Hamilton-Jacobi equation. The latter can be derived from the former in the following way. Using (5), (6) takes the explicit form

$$\frac{1}{2}\frac{\partial S_\mu}{\partial \phi}\frac{\partial S^\mu}{\partial \phi} + V + \partial_\mu S^\mu = 0. \tag{9}$$

Using the equation of motion (7), we write the first term in (9) as

$$\frac{1}{2}\frac{\partial S_\mu}{\partial \phi}\frac{\partial S^\mu}{\partial \phi} = \frac{1}{2}\frac{\partial S^0}{\partial \phi}\frac{\partial S^0}{\partial \phi} + \frac{1}{2}(\partial_i \phi)(\partial^i \phi), \tag{10}$$

where $i = 1, 2, 3$ are the space indices. Similarly, using (7) and (8), we write the last term in (9) as

$$\partial_\mu S^\mu = \partial_0 S^0 + d_i S^i - (\partial_i \phi)(\partial^i \phi). \tag{11}$$

Now introduce the quantity

$$\mathcal{S} = \int d^3x \, S^0, \tag{12}$$

so that

$$\frac{\partial S^0(\phi(x), x)}{\partial \phi(x)} = \frac{\delta \mathcal{S}([\phi(\mathbf{x}, t)], t)}{\delta \phi(\mathbf{x}; t)}, \tag{13}$$

where

$$\frac{\delta}{\delta \phi(\mathbf{x}; t)} \equiv \left. \frac{\delta}{\delta \phi(\mathbf{x})} \right|_{\phi(\mathbf{x}) = \phi(\mathbf{x}, t)} \tag{14}$$

is the space functional derivative. Thus, putting (10), (11), and (13) into (9) and integrating the resulting equation over d^3x, we obtain

$$\int d^3x \left[\frac{1}{2} \left(\frac{\delta \mathcal{S}}{\delta \phi(\mathbf{x}; t)} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right] + \partial_t \mathcal{S} = 0, \tag{15}$$

which is the standard noncovariant Hamilton-Jacobi equation (written for the time-dependent field $\phi(\mathbf{x}, t)$). The time evolution of the field $\phi(\mathbf{x}, t)$ is given by

$$\partial_t \phi(\mathbf{x}, t) = \frac{\delta \mathcal{S}}{\delta \phi(\mathbf{x}; t)}, \tag{16}$$

which is a consequence of the time component of (7). Note that in deriving (15) from (9), it was necessary to use the space part of the equations of motion (7). Whereas this fact does not play an important role in classical physics, it has far reaching consequences in the quantum case studied in Sec. 3.

2.2 Bohmian formulation of quantum field theory

Quantum field theory can be formulated in the functional Schrödinger picture as

$$\hat{H} \Psi = i\hbar \partial_t \Psi, \tag{17}$$

where, for the real scalar field ϕ,

$$\hat{H} = \int d^3x \left[-\frac{\hbar^2}{2} \left(\frac{\delta}{\delta \phi(\mathbf{x})} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) \right]. \tag{18}$$

By writing

$$\Psi([\phi(\mathbf{x})], t) = \mathcal{R}([\phi(\mathbf{x})], t) e^{i\mathcal{S}([\phi(\mathbf{x})], t)/\hbar}, \tag{19}$$

where $\mathcal{R}$ and $\mathcal{S}$ are real functionals, one finds that the complex equation (17) is equivalent to a set of two real equations

$$\int d^3x \left[\frac{1}{2} \left(\frac{\delta \mathcal{S}}{\delta \phi(\mathbf{x})} \right)^2 + \frac{1}{2} (\nabla \phi)^2 + V(\phi) + Q \right] + \partial_t \mathcal{S} = 0, \tag{20}$$

$$\int d^3x \left[\frac{\delta \mathcal{R}}{\delta \phi(\mathbf{x})} \frac{\delta \mathcal{S}}{\delta \phi(\mathbf{x})} + J \right] + \partial_t \mathcal{R} = 0, \tag{21}$$

where

$$Q = -\frac{\hbar^2}{2\mathcal{R}} \frac{\delta^2 \mathcal{R}}{\delta\phi^2(\mathbf{x})}, \tag{22}$$

$$J = \frac{\mathcal{R}}{2} \frac{\delta^2 \mathcal{S}}{\delta\phi^2(\mathbf{x})}. \tag{23}$$

Eq. (21) is also equivalent to

$$\partial_t \mathcal{R}^2 + \int d^3x \frac{\delta}{\delta\phi(\mathbf{x})} \left(\mathcal{R}^2 \frac{\delta \mathcal{S}}{\delta\phi(\mathbf{x})} \right) = 0. \tag{24}$$

Eq. (24) represents the unitarity of the theory, because it provides that the norm

$$\int [d\phi(\mathbf{x})] \, \Psi^* \Psi = \int [d\phi(\mathbf{x})] \, \mathcal{R}^2 \tag{25}$$

does not depend on time. The quantity $\mathcal{R}^2([\phi(\mathbf{x})], t)$ represents the probability density for fields to have the configuration $\phi(\mathbf{x})$ at time t. Instead of starting with (17), one can equivalently take Eqs. (20) and (21) as the starting point for quantization of fields. In addition, since Ψ must be a single-valued quantity, in the approach based on (20) and (21) one must require that the quantity $\exp i\mathcal{S}/\hbar$ should be single valued.

Eqs. (20) and (24) also suggest an interesting interpretation, known as the Bohmian interpretation of quantum field theory [13, 14, 15, 16]. The interpretation consists in the assumption that quantum fields have a deterministic time evolution given by the classical equation (16). Remarkably, the statistical predictions of this deterministic interpretation are equivalent to those of the conventional interpretation. In the deterministic interpretation, all quantum uncertainties are a consequence of the ignorance of the actual initial field configuration $\phi(\mathbf{x}, t_0)$. The main reason for the consistency of this interpretation is the fact that (24) with (16) represents the continuity equation, which provides that the statistical distribution $\rho([\phi(\mathbf{x})], t)$ of field configurations $\phi(\mathbf{x})$ is given by the quantum distribution $\rho = \mathcal{R}^2$ at *any* time t, provided that ρ is given by $\mathcal{R}^2$ at some initial time t_0. The initial distribution is arbitrary in principle, but a quantum H-theorem [27] explains why the quantum distribution is the most probable.

Comparing (20) with (15), we see that the quantum field satisfies an equation similar to the classical one, except for an additional quantum force resulting from the *nonlocal* quantum potential Q. (The nonlocality implies that the Bohmian hidden variable theory, with $\phi(\mathbf{x}, t)$ being the hidden variable, is not in contradiction with the Bell theorem that asserts that *local* hidden variables cannot be consistent with quantum mechanics.) The quantum equation of motion turns out to be

$$\partial^\mu \partial_\mu \phi + \frac{\partial V(\phi)}{\partial \phi} + \frac{\delta \mathcal{Q}}{\delta\phi(\mathbf{x}; t)} = 0, \tag{26}$$

where $\mathcal{Q} = \int d^3x \, Q$. The last term represents the deviation from the classical equation of motion.

The Bohmian interpretation may seem appealing to some, while unattractive to the others. An appealing feature is an explanation of the notorious "wave function collapse". An unattractive feature is the fact that the Bohmian interpretation is not covariant and requires the existence of a preferred Lorentz frame not determined by the theory, despite the fact that the statistical predictions obtained by averaging over hidden variables are Lorentz invariant [15]. However, at this level, nothing forces us to adopt this interpretation, just as nothing prevents us from adopting it. Adopting or rejecting this interpretation is more like a matter of taste. As we shall see in the next section, in the manifestly covariant formulation of quantum field theory based on the De Donder-Weyl formalism, the situation is quite different. There, one *must*

adopt a covariant version of the Bohmian equations of motion, because otherwise one cannot retain both covariance and consistency with the standard canonical quantization in Minkowski spacetime.

3 Covariant canonical quantization

Our basic idea for covariant canonical quantization is to find a quantum substitute for the classical covariant De Donder-Weyl Hamilton-Jacobi equation (9). For this purpose, we first formulate the classical De Donder-Weyl theory in a slightly different, more general way. Let $A([\phi],x)$ be a functional of $\phi(x)$ and a function of x. We define the derivative

$$\frac{dA([\phi],x)}{d\phi(x)} \equiv \int d^4x' \frac{\delta A([\phi],x')}{\delta\phi(x)}, \tag{27}$$

where $\delta/\delta\phi(x)$ is the spacetime functional derivative (not the space functional derivative in (14)). In particular, if $A([\phi],x)$ is a local functional, i.e., if $A([\phi],x) = A(\phi(x),x)$, then

$$\frac{dA(\phi(x),x)}{d\phi(x)} = \int d^4x' \frac{\delta A(\phi(x'),x')}{\delta\phi(x)} = \frac{\partial A(\phi(x),x)}{\partial\phi(x)}. \tag{28}$$

Thus we see that the derivative $d/d\phi$ is a generalization of the ordinary partial derivative $\partial/\partial\phi$, such that its action on nonlocal functionals is also well defined. An example of a particular interest is a functional nonlocal in space but local in time, so that

$$\frac{\delta A([\phi],x')}{\delta\phi(x)} = \frac{\delta A([\phi],x')}{\delta\phi(\mathbf{x};x^0)}\delta(x'^0 - x^0). \tag{29}$$

In this case, one can write

$$\frac{dA([\phi],x)}{d\phi(x)} = \frac{\delta}{\delta\phi(\mathbf{x};x^0)} \int d^3x' A([\phi],\mathbf{x}',x^0). \tag{30}$$

Being equiped with these mathematical tools, we can write (9) as

$$\frac{1}{2}\frac{dS_\mu}{d\phi}\frac{dS^\mu}{d\phi} + V + \partial_\mu S^\mu = 0, \tag{31}$$

which is the form appropriate for the quantum modification. Similarly, the classical equations of motion (7) can be written as

$$\partial^\mu\phi = \frac{dS^\mu}{d\phi}. \tag{32}$$

Now we are ready to propose a method of quantization that combines the classical covariant canonical De Donder-Weyl formalism with the standard time-space asymmetric canonical quantization of fields. Our starting point is the relation between the noncovariant classical Hamilton-Jacobi equation (15) and its quantum analog (20). Suppressing the time dependence of the field in (15), we see that they differ only in the existence of the Q-term in the quantum case. This suggests us to postulate the following quantum analog of the classical covariant equation (31):

$$\frac{1}{2}\frac{dS_\mu}{d\phi}\frac{dS^\mu}{d\phi} + V + Q + \partial_\mu S^\mu = 0. \tag{33}$$

Here $S^\mu = S^\mu([\phi],x)$ is a functional (not merely a function) of $\phi(x)$. This means that S^μ at x may depend on the field $\phi(x')$ at *all* points x'. Such spacetime nonlocalities absent in classical

physics are expected in quantum physics. Indeed, space nonlocalities appear in the conventional time-space asymmetric quantum field theory. Here, for the sake of covariance, we also allow the time nonlocalities (see also [19]). Thus Eq. (33) is manifestly covariant, provided that Q given by (22) can be written in a covariant form. The quantum equation (33) must be consistent with the conventional quantum equation (20). Indeed, by using a similar procedure to that used to show that (9) implies (15), one can show that (33) implies (20), provided that some additional conditions are fulfilled. First, S^0 must be local in time, so that (30) for $A = S^0$ can be used (compare with (13)). Second, S^i must be completely local, so that $dS^i/d\phi = \partial S^i/\partial\phi$, which implies

$$d_i S^i = \partial_i S^i + (\partial_i \phi)\frac{dS^i}{d\phi} \tag{34}$$

(compare with (8)). However, just as in the classical case, in this procedure it is *necessary* to use the space part of the equations of motion (7). Therefore, these classical equations of motion must be valid even in the quantum case. Since we want a covariant theory in which space and time play equal roles, the validity of the space part of the equations of motion (7) implies that their time part should also be valid. Consequently, in the covariant quantum theory based on the De Donder-Weyl formalism, one must require the validity of (32). This requirement is nothing but a covariant version of the Bohmian equation of motion, written for an arbitrarily nonlocal S^μ. (Note that, in order to achieve the consistency of the De Donder-Weyl quantization with the conventional quantization, the space part of the classical equations of motion have also been used in the approach of [11]. However, in that paper, the physical consequences of this fact have not been recognized.)

The next step is to find a covariant substitute for Eq. (21). For this purpose, we introduce a vector $R^\mu([\phi], x)$. The vector field R^μ can be viewed as generating a preferred foliation of spacetime, such that, in this foliation, the vector R^μ is normal to the leafs of that foliation. This allows us to introduce the quantity

$$\mathcal{R}([\phi], \Sigma) = \int_\Sigma d\Sigma_\mu R^\mu, \tag{35}$$

where Σ is a leaf (a 3-dimensional hypersurface) generated by R^μ. Similarly, a covariant version of (12) reads

$$\mathcal{S}([\phi], \Sigma) = \int_\Sigma d\Sigma_\mu S^\mu, \tag{36}$$

where Σ is generated by R^μ again. Consequenly, the covariant version of (19) reads

$$\Psi([\phi], \Sigma) = \mathcal{R}([\phi], \Sigma) e^{i\mathcal{S}([\phi],\Sigma)/\hbar}. \tag{37}$$

For R^μ we postulate the equation

$$\frac{dR^\mu}{d\phi}\frac{dS_\mu}{d\phi} + J + \partial_\mu R^\mu = 0. \tag{38}$$

In this way, a preferred foliation emerges dynamically, as a foliation generated by the solution R^μ of the equations (38) and (33). Note that R^μ does not play any role in classical physics, so the existence of a preferred foliation is a purely quantum effect. Now the relation between (38) and (21) is obtained by assuming that nature has chosen a solution of the form $R^\mu = (R^0, 0, 0, 0)$, where R^0 is local in time. In this case, by integrating (38) over d^3x and assuming again that S^0 is local in time, one obtains (21). Thus we see that (38) is a covariant substitute for (21).

It remains to write covariant versions of (22) and (23). They are simply

$$Q = -\frac{\hbar^2}{2\mathcal{R}}\frac{\delta^2 \mathcal{R}}{\delta_\Sigma \phi^2(x)}, \tag{39}$$

$$J = \frac{\mathcal{R}}{2} \frac{\delta^2 \mathcal{S}}{\delta_\Sigma \phi^2(x)}, \tag{40}$$

where $\delta/\delta_\Sigma\phi(x)$ is a version of (14) in which Σ is generated by R^μ. Here Σ depends on x (the point x is an element of Σ) and Σ is kept fixed in the variation $\delta_\Sigma\phi(x)$. Thus, (38) with (40) and (33) with (39) represent a covariant substitute for the functional Schrödinger equation (17) equivalent to (21) with (23) and (20) with (22).

The covariant Bohmian equations (32) imply a covariant version of (26)

$$\partial^\mu \partial_\mu \phi + \frac{\partial V}{\partial \phi} + \frac{dQ}{d\phi} = 0. \tag{41}$$

Since the last term can also be written as $\delta(\int d^4x Q)/\delta\phi(x)$, the equation of motion (41) can be obtained by varying the quantum action

$$\mathcal{A}_Q = \int d^4x \mathcal{L}_Q = \int d^4x (\mathcal{L} - Q). \tag{42}$$

To summarize, our covariant canonical quantization of fields is given by Eqs. (33), (38), (39), (40), and (32). The conventional functional Schrödinger equation corresponds to a special class of solutions of (33), (38), (39) and (40), for which $R^i = 0$, S^i are local, while R^0 and S^0 are local in time.

4 Generalizations

In this section we generalize the results obtained so far to include the cases of a larger number of fields, as well as the case of curved spacetime (that may be either a background spacetime or a dynamical spacetime).

Let $\phi(x) = \{\phi_a(x)\}$ be a collection of fields. We study a classical action (1), with $\mathcal{L}$ taking the form

$$\begin{aligned} \mathcal{L} &= \frac{1}{2} G^{ab}(\phi, x) g^{\mu\nu}(x) (\partial_\mu \phi_a)(\partial_\nu \phi_b) \\ &\quad + F^{a\mu}(\phi, x) \partial_\mu \phi_a - V(\phi, x). \end{aligned} \tag{43}$$

In particular, G^{ab}, $F^{a\mu}$, and V are proportional to $|g|^{1/2}$, where g is the determinant of the metric tensor $g_{\mu\nu}$. Thus the factor $|g|^{1/2}$ is included in the definition of $\mathcal{L}$, which makes the application of canonical methods easier. (The price we pay is that the general covariance is slightly less manifest.) We also introduce the quantity G_{ab} defined by

$$G_{ab} G^{bc} = \delta_a^c. \tag{44}$$

Thus G_{ab} is the matrix inverse to G^{ab}. (For the case in which the inverse does not exist, see below.) This allows us to consistently raise and lower the indices a, b with G^{ab} and G_{ab}, respectively. Since $G^{ab} \propto |g|^{1/2}$, we see that if $\partial_\mu \phi_a$ is a tensor, then $\partial^\mu \phi^a$ is a tensor density.

Now the canonical momenta are

$$\pi^{a\mu} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_a)} = \partial^\mu \phi^a + F^{a\mu}, \tag{45}$$

while the De Donder-Weyl Hamiltonian is

$$\begin{aligned} \mathcal{H} &= \pi^{a\mu} \partial_\mu \phi_a - \mathcal{L} \\ &= \frac{1}{2} (\partial^\mu \phi^a)(\partial_\mu \phi_a) + V \\ &= \frac{1}{2} \pi^{a\mu} \pi_{a\mu} - \pi^{a\mu} F_{a\mu} + \frac{1}{2} F^{a\mu} F_{a\mu} + V. \end{aligned} \tag{46}$$

The corresponding covariant canonical equations of motion are

$$\partial_\mu \phi_a = \frac{\partial \mathcal{H}}{\partial \pi^{a\mu}} = \pi_{a\mu} - F_{a\mu}, \tag{47}$$

$$\partial^\mu \pi_{a\mu} = -G_{ab}\frac{\partial \mathcal{H}}{\partial \phi_b} \equiv -\partial_a \mathcal{H}. \tag{48}$$

Here $\partial_a = G_{ab}\partial^b \neq \partial^b G_{ab}$, because G_{ab} depends on ϕ. The covariant Hamilton-Jacobi equations are

$$\pi^{a\mu} = \frac{\partial S^\mu}{\partial \phi_a} \equiv \partial^a S^\mu, \tag{49}$$

$$\frac{1}{2}(\partial^a S^\mu)(\partial_a S_\mu) - F_{a\mu}\partial^a S^\mu + \frac{1}{2}F^{a\mu}F_{a\mu} + V + \partial_\mu S^\mu = 0. \tag{50}$$

The total derivative is

$$d_\mu = \partial_\mu + (\partial_\mu \phi_a)\partial^a. \tag{51}$$

It is instructive to show explicitly that (50) is general covariant. It is not difficult to see that each term in (50) is proportional to $|g|^{1/2}$, i.e., that the left-hand side is a scalar density. In particular, S^μ is a vector density, so we write $S^\mu = |g|^{1/2}\tilde{S}^\mu$, where $\tilde{S}^\mu$ is a vector. To obtain a scalar equation, we multiply the whole equation with $|g|^{-1/2}$. The last term becomes $|g|^{-1/2}\partial_\mu(|g|^{1/2}\tilde{S}^\mu)$, which is nothing but the covariant derivative $\nabla_\mu \tilde{S}^\mu$. In a similar way, one can show that all other equations of this section are also general covariant.

The general covariant generalisation of the derivative (27) depends on the tensor nature of A. In our case A is a vector density A^μ, so (27) naturally generalizes to

$$\frac{dA^\mu([\phi], x)}{d\phi(x)} \equiv \frac{e^\mu_{\bar{\alpha}}(x)}{|g(x)|^{1/2}} \int d^4x'\, e^{\bar{\alpha}}_\nu(x') \frac{\delta A^\nu([\phi], x')}{\delta \phi(x)}. \tag{52}$$

Here $e^\mu_{\bar{\alpha}}$ is the tetrad satisfying $e^\mu_{\bar{\alpha}} e^{\bar{\alpha}\nu} = g^{\mu\nu}$, where $\bar{\alpha}$ is an index in the internal $SO(1,3)$ group.

Now the quantization is straightforward. In (50), one replaces the derivative ∂^a with the derivative $d^a = d/d\phi_a$ and adds the Q-term. The quantum potential is

$$Q = -\frac{\hbar^2}{2\mathcal{R}} \frac{\delta}{\delta_\Sigma \phi_a} G_{ab} \frac{\delta}{\delta_\Sigma \phi_b} \mathcal{R}. \tag{53}$$

Eq. (38) generalizes to

$$(d^a R^\mu)(d_a S_\mu) - F_{a\mu} d^a R^\mu + J + \partial_\mu R^\mu = 0, \tag{54}$$

where

$$J = \frac{\mathcal{R}}{2} \frac{\delta}{\delta_\Sigma \phi_a} \left(G_{ab} \frac{\delta \mathcal{S}}{\delta_\Sigma \phi_b} - F_{a\mu} r^\mu \right), \tag{55}$$

and $r^\mu = R^\mu/(R^\lambda R_\lambda)^{1/2}$. The particular orderings in (53) and (55) are chosen so that they lead to a Schrödinger equation with a hermitian Hamilton operator. Eqs. (35) and (36) now take the manifestly covariant form

$$\mathcal{R}([\phi], \Sigma) = \int_\Sigma d\Sigma_\mu \tilde{R}^\mu, \tag{56}$$

$$\mathcal{S}([\phi], \Sigma) = \int_\Sigma d\Sigma_\mu \tilde{S}^\mu, \tag{57}$$

where $\tilde{S}^\mu$ and $\tilde{R}^\mu = R^\mu/|g|^{1/2}$ are vectors. The wave functional is again given by (37) and the Bohmian equations of motion

$$\partial_\mu \phi_a = d_a S_\mu - F_{a\mu} \tag{58}$$

are equivalent to the equations obtained by varying the quantum action (42).

The formalism can be further generalized to the case in which $G^{ab}g^{\mu\nu}$ in (43) is replaced with a more general quantity of the form $G^{ab\mu\nu}$. This, of course, does not present any problem for the classical formalism. The quantum equation (54) generalises to

$$G_{ab\mu\nu}\frac{dR^{\mu}}{d\phi_a}\frac{dS^{\nu}}{d\phi_b} - G_{ab\mu\nu}F^{b\nu}\frac{dR^{\mu}}{d\phi_a} + J + \partial_{\mu}R^{\mu} = 0, \qquad (59)$$

where $G_{ab\mu\nu}$ is the inverse of $G^{ab\mu\nu}$, in the sense that

$$G^{ab'\mu\nu'}G_{b'b\nu'\nu} = \delta^a_b\delta^{\mu}_{\nu}. \qquad (60)$$

The generalizion of (53) and (55) consists in the replacement of G_{ab} in (53) and (55) with $G^{(r)}_{ab}$, where

$$G^{(r)}_{ab} = G_{ab\mu\nu}r^{\mu}r^{\nu}. \qquad (61)$$

By considering the case $R^{\mu} = (R^0, 0, 0, 0)$, it is easy to see that this generalization leads to the usual Schrödinger equation.

In some cases, such as gauge theories, the invers of $G^{ab\mu\nu}$ does not exist. In such cases, one can define $G^{ab\mu\nu}$ and its inverse $G_{ab\mu\nu}$ by introducing a gauge fixing term or by using other tricks [8, 28, 29].

Our formalism leads to particularly interesting consequences when applied to quantum gravity and other theories with reparametrization invariance. We study this in more detail in the next section.

5 Reparametrization-invariant theories

5.1 A toy model

Consider a system with two degrees of freedom $\phi_1(t)$ and $\phi_2(t)$. Let the action be given by

$$A = \int dt\, L = \int dt\, \phi_2\left[\frac{\dot{\phi_1}^2}{2(\phi_2)^2} - \tilde{V}(\phi_1)\right], \qquad (62)$$

where the dot denotes the time derivative d_t. The action is invariant with respect to time reparametrizations $t \to t'(t)$, $\phi_2 \to \phi'_2 = (dt/dt')\phi_2$. As is well known, such theories lead to a Hamiltonian constraint and thus serve as toy models instructive for an easier understanding of some of the peculiar properties of classical and quantum gravity [30, 31]. Here we study this model by using the Hamilton-Jacobi formalism. Since there is no space (but only time) in this model, the De Donder-Weyl canonical formalism is identical to the conventional classical canonical formalism.

The canonical momenta are

$$\pi^1 = \frac{\partial L}{\partial\dot{\phi_1}} = \frac{\dot{\phi_1}}{\phi_2}, \qquad \pi^2 = \frac{\partial L}{\partial\dot{\phi_2}} = 0. \qquad (63)$$

Since $\pi^2 = 0$, the Hamiltonian is

$$H = \pi^1\dot{\phi_1} - L = \phi_2\left(\frac{(\pi^1)^2}{2} + \tilde{V}\right). \qquad (64)$$

The Hamilton-Jacobi formalism is given by the equations of motion

$$\pi^1 = \frac{\partial S}{\partial \phi_1} = \frac{\dot{\phi}_1}{\phi_2}, \quad \pi^2 = \frac{\partial S}{\partial \phi_2} = 0, \tag{65}$$

together with the Hamilton-Jacobi equation

$$\phi_2 \left[\frac{1}{2} \left(\frac{\partial S}{\partial \phi_1} \right)^2 + \tilde{V} \right] + \partial_t S = 0. \tag{66}$$

From the second equation in (65), we see that S does not depend on ϕ_2. Consequently, by applying the derivative $\partial/\partial\phi_2$ to (66), one obtains

$$\frac{1}{2} \left(\frac{\partial S}{\partial \phi_1} \right)^2 + \tilde{V} = 0, \tag{67}$$

which is nothing but the Hamiltonian constraint $H = 0$. It is equivalent to the equation of motion that one obtains by varying (62) with respect to ϕ_2. Comparing (67) with (66), one also finds

$$\partial_t S = 0. \tag{68}$$

Applying the derivative $\partial/\partial\phi_1$ to (67), one obtains

$$\frac{\partial S}{\partial \phi_1} \frac{\partial}{\partial \phi_1} \frac{\partial S}{\partial \phi_1} + \frac{\partial \tilde{V}}{\partial \phi_1} = 0. \tag{69}$$

Using (65) and the fact that

$$\dot{\phi}_1 \frac{\partial}{\partial \phi_1} \frac{\partial S}{\partial \phi_1} = d_t \frac{\partial S}{\partial \phi_1}, \tag{70}$$

(69) implies

$$d_t \left(\frac{\dot{\phi}_1}{\phi_2} \right) + \frac{\partial \tilde{V}}{\partial \phi_1} = 0. \tag{71}$$

This is nothing but the equation of motion obtained by varying ϕ_1 in (62).

Now consider the quantization. Following the general method developed is Sec. 4, we introduce the quantum potential

$$Q = -\frac{\hbar^2}{2R} \phi_2 \frac{d^2 R}{d\phi_1^2}, \tag{72}$$

and replace (66) with

$$\phi_2 \left[\frac{1}{2} \left(\frac{dS}{d\phi_1} \right)^2 + \tilde{V} \right] + Q + \partial_t S = 0. \tag{73}$$

Eqs. (65) are valid with the derivatives $\partial/\partial\phi_a$ replaced with $d/d\phi_a$. The conservation equation reads

$$\partial_t R^2 + \frac{d}{d\phi_1} \left(R^2 \phi_2 \frac{dS}{d\phi_1} \right) = 0. \tag{74}$$

Applying the derivative $d/d\phi_2$ to (73), we obtain

$$\frac{1}{2} \left(\frac{dS}{d\phi_1} \right)^2 + \tilde{V} + \tilde{Q} + \phi_2 \frac{d\tilde{Q}}{d\phi_2} = 0, \tag{75}$$

where $Q = \phi_2 \tilde{Q}$. Combining (73) and (75), we obtain

$$\partial_t S - (\phi_2)^2 \frac{d\tilde{Q}}{d\phi_2} = 0. \tag{76}$$

Eqs. (75) and (76) are the quantum analogs of (67) and (68), respectively.

Now compare the results above with the conventional method of quantization of the Hamiltonian constraint, based on the Wheeler-DeWitt equation $\hat{H}\Psi = 0$. By writing $\Psi = R\exp(iS/\hbar)$, one obtains

$$\frac{1}{2}\left(\frac{dS}{d\phi_1}\right)^2 + \tilde{V} + \tilde{Q} = 0, \tag{77}$$

$$\partial_t S = 0, \tag{78}$$

and

$$\frac{d}{d\phi_1}\left(R^2 \frac{dS}{d\phi_1}\right) = 0, \tag{79}$$

instead of (75), (76), and (74), respectively. In particular, we see that S and R are time independent, which corresponds to the well-known problem of time in quantum gravity [31, 32, 33], consisting in the fact that the wave function(al) Ψ does not depend on time. In our approach, $\partial_t S \neq 0$ and $\partial_t R \neq 0$ in the general quantum case, so there is no problem of time. However, our quantization contains the Wheeler-DeWitt quantization as a special case. If R is a time-independent solution, then (74) reduces to (79). Consequently, R may be a ϕ_2-independent solution of (74). If R does not depend on ϕ_2, then (72) implies $d\tilde{Q}/d\phi_2 = 0$. Consequently, (75) and (76) reduce to (77) and (78), respectively.

It is also instructive to show explicitly that our method of quantization leads to the Bohmian equations of motion that can be derived from the Lagrangian $L_Q = L - Q$, or equivalently, from the Hamiltonian

$$H_Q = H + Q. \tag{80}$$

We have

$$\begin{aligned} \dot{\phi}_1 &= \frac{dH_Q}{d\pi^1} = \phi_2 \pi^1, \\ \dot{\pi}^1 &= -\frac{dH_Q}{d\phi_1} = -\phi_2 \left(\frac{\partial \tilde{V}}{\partial \phi_1} + \frac{d\tilde{Q}}{d\phi_1}\right), \end{aligned} \tag{81}$$

$$\begin{aligned} \dot{\phi}_2 &= \frac{dH_Q}{d\pi^2} = 0, \\ 0 = \dot{\pi}^2 &= -\frac{dH_Q}{d\phi_2} = \frac{(\pi^1)^2}{2} + \tilde{V} + \tilde{Q} + \phi_2 \frac{d\tilde{Q}}{d\phi_2}. \end{aligned} \tag{82}$$

The second equation in (82) is equivalent to (75). Therefore, the only nontrivial Hamilton equation that remains to be proved in the quantum Hamilton-Jacobi framework is the second equation in (81). Applying the derivative $d/d\phi_1$ to (73) and using (65) and

$$\partial_t \frac{dS}{d\phi_1} + \dot{\phi}_1 \frac{d}{d\phi_1}\frac{dS}{d\phi_1} = d_t \frac{dS}{d\phi_1} \tag{83}$$

(compare with (70)), one obtains

$$d_t \pi^1 + \phi_2 \left(\frac{\partial \tilde{V}}{\partial \phi_1} + \frac{d\tilde{Q}}{d\phi_1}\right) = 0, \tag{84}$$

which is the second equation in (81).

Finally, note that (73) and (74) can also be derived from the Schrödinger equation

$$\hat{H}\Psi = i\hbar\partial_t\Psi. \tag{85}$$

At first sight, this seems to be inconsistent with the classical Hamiltonian constraint $H = 0$. However, in the classical limit $\hbar \to 0$, (85) leads to (66), which, as we have seen, *does* lead to the Hamiltonian constraint (67). Thus, we have a remarkable result that (85) *is* consistent with $H = 0$, provided that the Bohmian equation corresponding to the second equation in (65) is valid.

5.2 Quantum gravity

In this subsection, we sketch the main points relevant for the application to quantum gravity.

The classical gravitational action is

$$\mathcal{A} = \int d^4x |g|^{1/2} R, \tag{86}$$

where R is the scalar curvature. To write the Lagrangian in a form appropriate for a canonical treatment, we write

$$|g|^{1/2} R = \frac{1}{2} G^{\alpha\beta\mu\gamma\delta\nu} (\partial_\mu g_{\alpha\beta})(\partial_\nu g_{\gamma\delta}) + \text{total derivative}, \tag{87}$$

and ignore the total-derivative term. The quantity $G^{\alpha\beta\mu\gamma\delta\nu}$ and its inverse $G_{\alpha\beta\mu\gamma\delta\nu}$ depend on $g_{\alpha\beta}$ but not on the derivatives of $g_{\alpha\beta}$ [28, 34, 35]. The fields ϕ_a are the components of the metric $g_{\alpha\beta}$. The essential property of our covariant canonical quantization is that all 10 components $g_{\alpha\beta}$ are quantized, in contrast to the conventional noncovariant canonical quantization where only the space components g_{ij} are quantized. Following the general method developed in Sec. 4, one finds the following quantum equations:

$$\frac{1}{2} G_{\alpha\beta\mu\gamma\delta\nu} \frac{dS^\mu}{dg_{\alpha\beta}} \frac{dS^\nu}{dg_{\gamma\delta}} + Q + \partial_\mu S^\mu = 0, \tag{88}$$

$$Q = -\frac{\hbar^2}{2\mathcal{R}} \frac{\delta}{\delta_\Sigma g_{\alpha\beta}} G^{(r)}_{\alpha\beta\gamma\delta} \frac{\delta}{\delta_\Sigma g_{\gamma\delta}} \mathcal{R}, \tag{89}$$

$$G_{\alpha\beta\mu\gamma\delta\nu} \frac{dR^\mu}{dg_{\alpha\beta}} \frac{dS^\nu}{dg_{\gamma\delta}} + J + \partial_\mu R^\mu = 0, \tag{90}$$

$$J = \frac{\mathcal{R}}{2} \frac{\delta}{\delta_\Sigma g_{\alpha\beta}} G^{(r)}_{\alpha\beta\gamma\delta} \frac{\delta}{\delta_\Sigma g_{\gamma\delta}} \mathcal{S}, \tag{91}$$

where

$$G^{(r)}_{\alpha\beta\gamma\delta} = G_{\alpha\beta\mu\gamma\delta\nu} r^\mu r^\nu. \tag{92}$$

The Bohmian equations of motion

$$\partial_\mu g_{\alpha\beta} = G_{\alpha\beta\mu\gamma\delta\nu} \frac{dS^\mu}{dg_{\gamma\delta}} \tag{93}$$

are equivalent to the equations of motion obtained by varying the quantum action

$$\mathcal{A}_Q = \int d^4x (|g|^{1/2} R - Q). \tag{94}$$

This leads to the equation of motion

$$R^{\mu\nu} - \frac{g^{\mu\nu}}{2} R + |g|^{-1/2} \frac{dQ}{dg_{\mu\nu}} = 0. \tag{95}$$

The potential Q is a scalar density, so we can write $Q = |g|^{1/2} \tilde{Q}$, where $\tilde{Q}$ is a scalar. Consequently, (95) can be written as

$$R^{\mu\nu} + \frac{d\tilde{Q}}{dg_{\mu\nu}} - \frac{g^{\mu\nu}}{2} (R - \tilde{Q}) = 0. \tag{96}$$

Another suggestive form is

$$\frac{g^{\mu\nu}}{2} R - R^{\mu\nu} = 8\pi G_N T^{\mu\nu}, \tag{97}$$

where

$$T^{\mu\nu} = \frac{1}{16\pi G_N} \left(2 \frac{d\tilde{Q}}{dg_{\mu\nu}} + g^{\mu\nu} \tilde{Q} \right). \tag{98}$$

Note that (97) and (98) imply that the Bohmian equations of motion are fully covariant. By contrast, if the quantization of gravity is based on the conventional canonical Wheeler-DeWitt equation that does *not* treat space and time on an equal footing, then the Bohmian interpretation leads to an equation similar to (97), but with a noncovariant energy-momentum tensor of the form [36]

$$T^{ij} \propto \frac{d\tilde{Q}}{dg_{ij}}, \quad T^{0\mu} \propto \tilde{Q} g^{0\mu}. \tag{99}$$

We also note that, similarly to the toy model studied in the preceding subsection, the conventional Wheeler-DeWitt quantization correponds to a special case in which $R^i = 0$, S^i are local, while S^0 and R^0 are functionals which are local in time and do not depend on $g_{0\mu}$ and x^0.

The functionals S^μ and R^μ (that determine also Ψ) describing the quantum state are functionals of the metric $g_{\alpha\beta}$. However, the theory is covariant, so one expects that physical results should not depend on the choice of coordinates, but only on the 4-geometry. In other words, the theory is expected to be invariant with respect to active 4-diffeomorhisms. At the moment, we do not know how to incorporate this property explicitly. However, if this 4-diffeomorhism invariance is explicitly realized, then one expects that ultraviolet divergences should be absent [3]. Perhaps this could be realized explicitly by introducing a time-space symmetric version of loop quantum gravity based on the spacetime covariant Ashtekar variables [37].

6 Discussion

The theory we have proposed in this paper offers a solution to several fundamental problems, but also rises some new problems.

First, the theory offers a manifestly covariant method of field quantization, based on the classical De Donder-Weyl formalism. The method treats space and time on an equal footing. Unlike the conventional canonical quantization, it is not formulated in terms of a single complex Schrödinger-like equation, but in terms of two coupled real equations. (In such a formulation, operators and commutation relations do not play any fundamental role.) Nevertheless, if the solution satisfies certain additional conditions, then the solution satisfies the conventional functional Schrödinger equation. These conditions are $R^i = 0$, locality of S^i, and locality in time of R^0 and S^0. These conditions are clearly not time-space symmetric. Since the predictions of the Schrödinger equation in Minkowski spacetime are in agreement with experiments, one would like

to explain why nature chooses solutions that, at least approximately, satisfy these conditions. Our theory does not explain that. However, in our theory, the observed time-space asymmetry of quantum field theory becomes a problem analogous to the observed time-space asymmetry in cosmology (at large scales, the universe is homogeneous and isotropic in space but not in time), or to the observed time-space asymmetry of thermodynamics (the entropy increases with time but not with space). Nothing prevents solutions that obey the observed quantum, cosmological, or thermodynamical time-space asymmetric rules, but a compelling explanation of these rules is missing. This suggests that the observed quantum time-space asymmetry might be of the cosmological origin. Another possibility is that our covariant quantum theory should be reformulated such that the direction of R^{μ} fully determines the directions in which the relevant quantities are local/nonlocal, but this would make the theory less elegant and perhaps would suppress interesting nonlocal effects that might exist in nature. (For example, such nonlocal effects might play a role for solving the black-hole information paradox.)

Second, the theory offers a solution to the problem of time in quantum gravity. In general, in our approach to quantum gravity, the functionals S^{μ} and R^{μ} depend on both space and time. The corresponding wave functional Ψ depends on the hypersurface. Nevertheless, in the classical limit, the Hamiltonian constraint is always valid. In addition, if the quantum state satisfies certain additional conditions, then it satisfies the conventional Wheeler-DeWitt equation (with a fixed ordering of operators!).

Third, the theory offers a covariant version of Bohmian mechanics, with time and space treated on an equal footing. The covariance of Bohmian mechanics is a direct consequence of the manifest covariance of the quantization procedure. Note that a frequent argument against the previous versions of Bohmian mechanics is their dependence on the choice of the time coordinate, even when the predictions of the conventional interpretation of quantum field theory do not depend on this choice. In these previous versions of Bohmian mechanics, one has to choose a preferred foliation of spacetime in a more or less ad hoc way (for a recent interesting attempt, see [38]). In our approach, the preferred foliation is generated dynamically by R^{μ}. This quantity does not play any role in classical physics, so the preferred foliation is a purely quantum effect. However, the Bohmian equations of motion themselves have a manifestly covariant form and do not depend explicitly on R^{μ}.

Fourth, our theory offers a new reason why one should adopt the Bohmian interpretation. Of course, our covariant method of quantization by itself, just as any other method of quantization, does not automatically imply the Bohmian interpretation. However, the need for the Bohmian interpretation emerges from the requirement that our covariant method should be consistent with the conventional noncovariant method. Those who, for some personal reasons, do not like the Bohmian deterministic interpretation, may take this as a problem of our quantization method. In our view, this result (see also the heuristic arguments presented in the Introduction) suggests that Bohmian mechanics might not be just one of interpretations, but a part of the formalism without which the covariant quantum theory cannot be formulated consistently. Note also that the adoption of Bohmian mechanics automatically removes uncertainties about the interpretational issues of quantum theory.

We also note that, in general, R^{μ} does not need to be timelike, so the preferred foliation does not need to be a foliation into spacelike hypersurfaces. This implies that our covariant canonical theory is able to deal even with spacetimes that are not globally hyperbolic.

The existence of a dynamically generated preferred foliation may also play an important role in semiclassical gravity, especially for the problem of definition of particles. This is because the definition of particles in the conventional semiclassical gravity depends on the choice of time [39].

A problem we have not discussed in this paper is how to describe fermionic fields within the

covariant Bohmian framework. In fact, a "standard" Bohmian interpretation of fermionic fields does not yet exist even within the conventional canonical quantization. However, the work on this issue is in progress.

To conclude, we believe that the quantization based on De Donder-Weyl covariant canonical formalism is an interesting idea worthwhile of further investigation. The main advantage is the manifest covariance with space and time treated on an equal footing. Another advantage is the lack of the interpretational ambiguities because the Bohmian interpretation emerges automatically. The main problem seems to be the locality/nonlocality issue. The approach of [11] appears to be too local, whereas the approach of the present paper allows nonlocalities that are not allowed by the conventional noncovariant Schrödinger equation. This may mean that the whole idea of quantization based on the De Donder-Weyl formalism is wrong, or that it has to be reformulated, or that the nonlocalities absent in the conventional noncovariant quantization correspond to new genuine physical effects.

Acknowledgements. This work was supported by the Ministry of Science and Technology of the Republic of Croatia under Contract No. 0098002.

References

[1] M.B. Green, J.H. Schwarz, E. Witten, Superstring Theory, Cambridge University Press, Cambridge (1987)

[2] M. Gaul, C. Rovelli, Lect. Notes Phys. **541** (2000) 277

[3] T. Thiemann, Lect. Notes Phys. **631** (2003) 41

[4] C. Rovelli, Quantum Gravity, Cambridge University Press, Cambridge, (2004); http://www.cpt.univ-mrs.fr/~rovelli

[5] T. Banks, W. Fischler, S.H. Shenker, L. Suskind, Phys. Rev. D **55** (1997) 5112

[6] Č. Crnković, E. Witten, in: S.W. Hawking, W. Israel (Eds.), Three Hundred Years of Gravitation, Cambridge University Press, Cambridge (1987)

[7] Č. Crnković, Class. Quant. Grav. **5** (1988) 1557

[8] H.A. Kastrup, Phys. Rep. **101** (1983) 1

[9] M.J. Gotay, J. Isenberg, J.E. Marsden, physics/9801019

[10] C. Rovelli, gr-qc/0207043

[11] I.V. Kanatchikov, Phys. Lett. A **283** (2001) 25

[12] I.V. Kanatchikov, Int. J. Theor. Phys. **40** (2001) 1121

[13] D. Bohm, Phys. Rev. **85** (1952) 180

[14] D. Bohm, B.J. Hiley, P.N. Kaloyerou, Phys. Rep. **144** (1987) 349

[15] P.R. Holland, Phys. Rep. **224** (1993) 95

[16] P.R. Holland, The Quantum Theory of Motion, Cambridge University Press, Cambridge (1993)

[17] D.F. Styer et al., Am. J. Phys. **70** (2002) 288

[18] H. Nikolić, quant-ph/0307179

[19] H. Nikolić, quant-ph/0406173, to appear in Found. Phys. Lett.

[20] J.A. deBarros, N. Pinto-Neto, M.A. Sagioro-Leal, Phys. Lett. A **241** (1998) 229

[21] R. Colistete Jr., J.C. Fabris, N. Pinto-Neto, Phys. Rev. D **57** (1998) 4707

[22] N. Pinto-Neto, R. Colistete Jr., Phys. Lett. A **290** (2001) 219

[23] J. Marto, P.V. Moniz, Phys. Rev. D **65** (2001) 023516

[24] N. Pinto-Neto, E.S. Santini, Phys. Lett. A **315** (2003) 36

[25] G.D. Barbosa, N. Pinto-Neto, Phys. Rev. D **69** (2004) 065014

[26] J. von Rieth, J. Math. Phys. **25** (1984) 1102

[27] A. Valentini, Phys. Lett. A **156** (1991) 5

[28] P. Hořava, Class. Quant. Grav. **8** (1991) 2069

[29] I.V. Kanatchikov, Rept. Math. Phys. **53** (2004) 181

[30] K. Kuchař, in: W. Israel (Ed.), Relativity, Astrophysics, and Cosmology, D. Reidel Publishing Company, Dordrecht/Boston (1973)

[31] T. Padmanabhan, Int. J. Mod. Phys. A **4** (1989) 4735

[32] K. Kuchař, in: Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics, World Scientific, Singapore (1992)

[33] C.J. Isham, gr-qc/9210011

[34] J.D.E. Grant, I.G. Moss, Phys. Rev. D **56** (1997) 6284

[35] T. Padmanabhan, Phys. Rep. **406** (2005) 49

[36] A. Shojai, F. Shojai, Class. Quant. Grav. **21** (2004) 1

[37] G. Esposito, G. Gionti, C. Stornaiolo, Nuovo Cim. **B110** (1995) 1137

[38] G. Horton, C. Dewdney, J. Phys. A **37** (2004) 11935

[39] N.D. Birrell, P.C.W. Davies, Quantum Fields in Curved Space, Cambridge Press, NY (1982)